1-7 Absolute Value Equations & Inequalities

Essential Question: Why does the solution for an absolute value equation or inequality typically result in a pair of equations or inequalities?

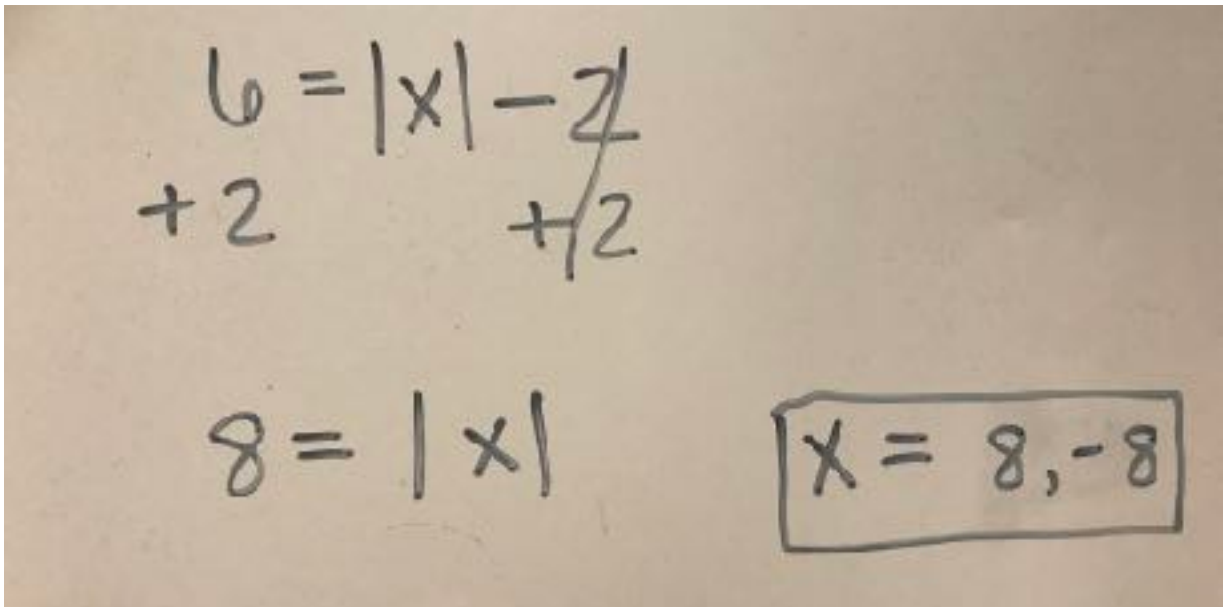
Example 1: Understand Absolute Value Equations

The absolute value of a number is its distance from 0 on a number line.

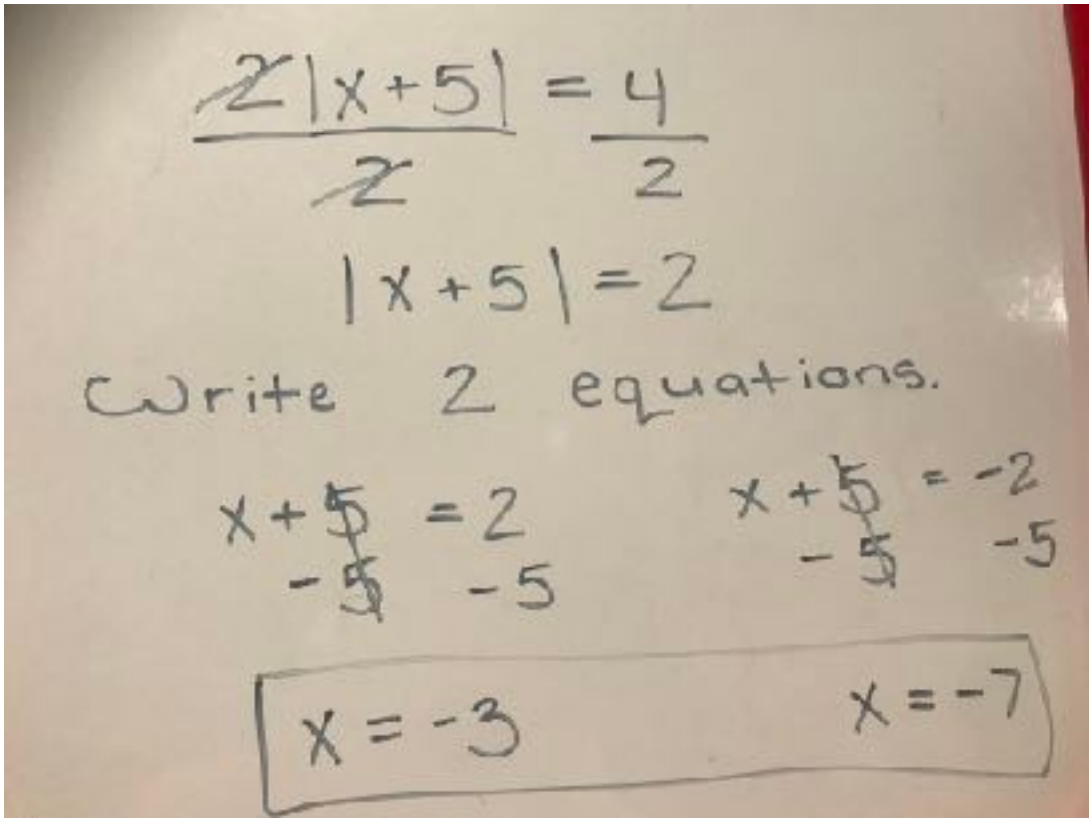
Example: |3| is 3 and |-3| is 3. <u>Remember an absolute value is always positive!</u>

Steps:
1. Isolate the absolute value expression.
2. Set the quantity inside the absolute value notation equal to + and - the quantity on the other side of the equation.
3. Solve for the unknown in both equations.
4. Check your answer.

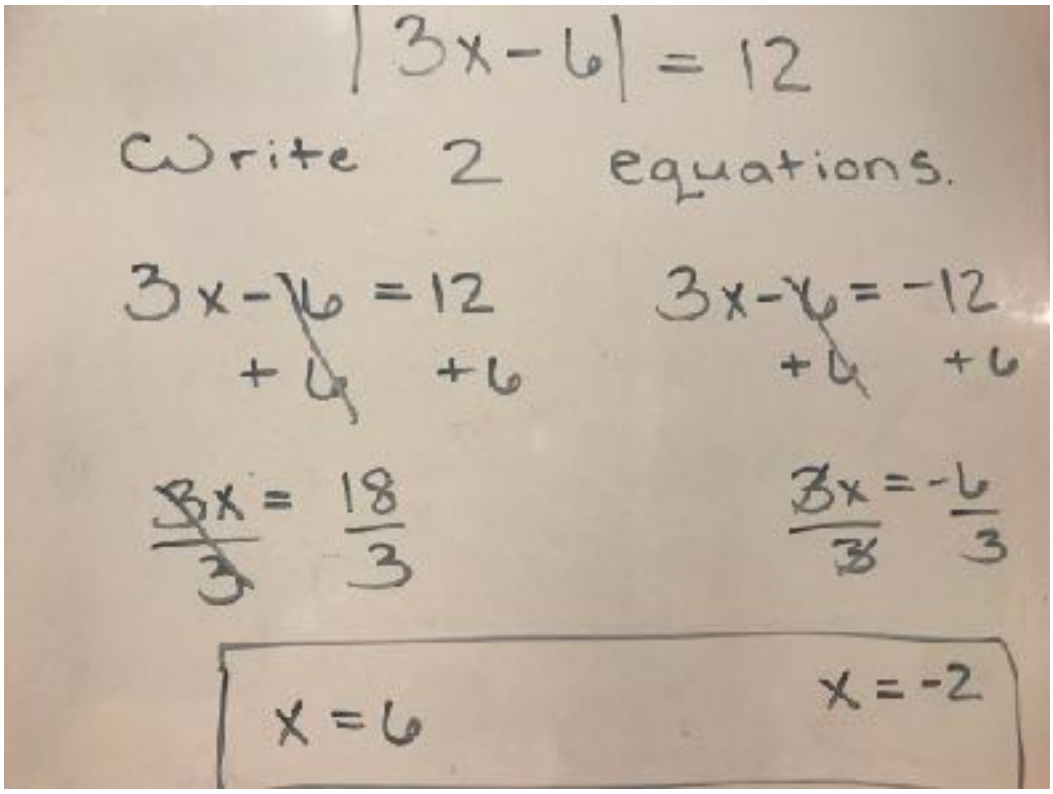
1. 6 = |x| - 2



2. $2|x + 5| = 4$



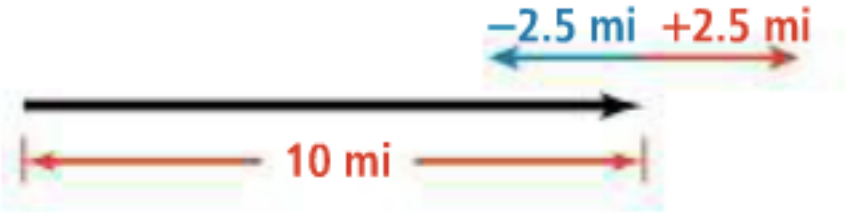
3. $|3x - 6| = 12$



Example 2: Apply an Absolute Value Equation

1. Write & solve an absolute value equation for the minimum and maximum times for an object moving at the given speed to travel the given distance.

25. 5 mi/h



|5x -10| = 2.5

Minimum time: 5x - 10 = -2.5 —> 5x -10 + 10 = -2.5 + 10 —> 5x = 7.5 —> x = 1.5 h

Maximum time: 5x -10 = 2.5 —> 5x -10 + 10 = 2.5 + 10 —> 5x = 12.5 —> x = 2.5 h

Example 3: Understand Absolute Value Inequalities

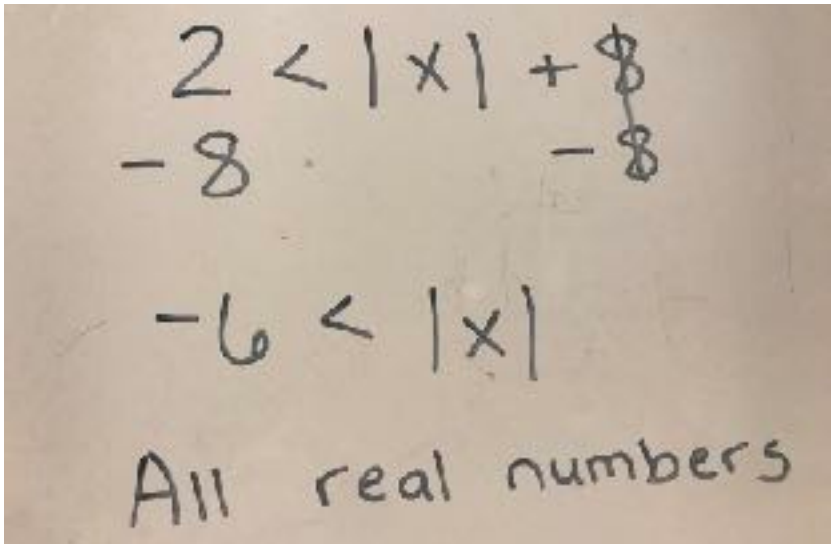
Steps:

1. Isolate the absolute value expression on the left side of the inequality.
2. If the number on the other side of the inequality sign is negative, your equation either has no solution or all real numbers as solutions. Use the sign of each side of your inequality to decide which of these cases holds. If the number on the other side of the inequality sign is positive, proceed to step 3.
3. Remove the absolute value bars by setting up a compound inequality. The type of inequality sign in the problem will tell us how to set up the compound inequality.
    - If your problem has a **greater than** or **greater than or equal to sign** (your problem now says that an absolute value is greater than a number), then set up an "or" compound inequality that looks like this: (quantity inside absolute value) < -(number on other side) OR (quantity inside absolute value) > (number on other side)
    - If your absolute value is **less than** or **less than or equal to** a number, then set up a three-part compound inequality that looks like this: -(number on other side) < (quantity inside absolute value) < (number on other side)
4. Solve the inequalities.

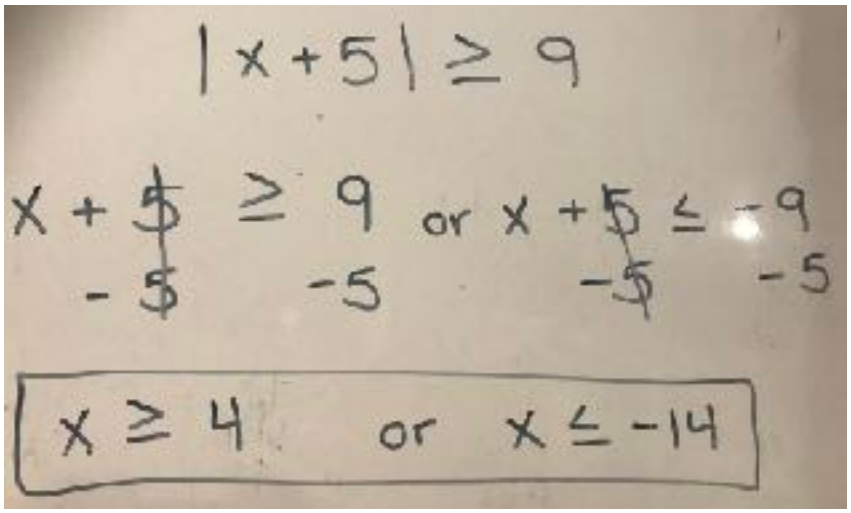
Cheat Sheet

p = a positive number	
$\|x\| = p$	$x = p, -p$
$\|x\| < p$	$x > -p$ and $x < p$ (Can also write as — $p < x < p$)
$\|x\| \leq p$	$x \geq -p$ and $x \leq p$ (Can also write as — $p \leq x \leq p$)
$\|x\| > p$	$x < -p$ or $x > p$
$\|x\| \geq p$	$x \leq -p$ or $x \geq p$
n = a negative number	
$\|x\| = n$	No solution
$\|x\| < n$	No solution
$\|x\| \leq n$	No solution
$\|x\| > n$	All real numbers
$\|x\| \geq n$	All real numbers

1. $2 < |x| + 8$



2. $|x + 5| \geq 9$



3. $|4x - 12| \leq 20$